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Reliability Analysis of Isolated Column Footings using Monte Carlo Simulation

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ABSTRACT- Uncertainties are present in all parameters involved in the design of structures. Owing to the complexities associated with handling random parameters, a deterministic approach is generally employed in the design of structures. Appropriately selected safety factors are used to compensate for the effect of uncertainties. It is impossible to estimate the safety margins of the resulting design using deterministic approaches. Reliability analysis provides a rational approach to evaluate the safety of structures in presence of uncertainties. The present paper deals with the reliability analysis of an isolated reinforced concrete footing, designed using the limit state method. Uncertainties associated with material properties, geometric properties and loading are considered and the parameters are modeled as normally/uniformly distributed random variables. Reliability analysis is carried out using Monte Carlo simulations using codes developed in Matlab. The probabilities of failure for different failure modes are estimated. The variation of reliability index with load factor as well as age of the structure is also studied.The paper demonstrates the usefulness of Monte Carlo simulations in obtaining probabilistic information for problems involving random parameters.

INDEX TERMS –Failure modes ,Isolated footing, Monte Carlo Simulation,Random variables, Reliability analysis, Reliability Index, Structural Reliability

1INTRODUCTION

Uncertainties are present in all real world problems. Due to presence of uncertainties in the various parameters required for the analysis and design of structures, it is very difficult to measure the absolute safety of structures using deterministic analysis. Therefore, one of the most important ways to specify arational criterion for ensuring the safety of the structure is its reliability or probability of failure [1].

Engineering community, building users and owner of building always expects structure and its foundation to be designed with a reasonably safety margin. In practices, these expectations are achieved by following the provisions in the design codes which is based on experience, practice and judgment. However, this approach lacks systematic basis for evaluating the degree of conservativeness and may result inadequate or uneconomical designs. To assess the safety and to enforce the safety margins, it is essential to characterize and include all major sources of uncertainties associated with the analysis and design of structural systems [6]. Reliability based design should be used for the new structures and the partial safety factor corresponding to reliability can be used for the design.

Presently, Canada, United State of America, United Kingdom and Norway follows the reliability based design of structure [1].Reliability analysis plays significant role in case of an existing structure too when the function of the building is changed.

The different methods of reliability analysis are First order Reliability Method (FORM), Second order Reliability Method (SORM), Support Vector Machine (SVM) method and Monte Carlo Simulation (MCS). Though computationally expensive, MCS method gives very accurate results and hence the method attracted many researchers [7], [9].

It is well known that the uncertainties in foundation are much more than that of super structures.Strength of materials, loads on foundation, bearing capacity of soil and size of the footing are some of the uncertain parameters associated with design of foundation. Youssef Abdel Massih et al. [12] presented a reliability based approach for analysis and design of the strip footing subjected to vertical load with or without pseudo-static seismic loading and the bearing failure was taken as the critical mode of failure. The differential settlement of footings on cohesionless soil was analysed by deriving the reliability based design equation for serviceability limit state considering allowable angular distortion, site variability and footing spacing as the key parameters [11]. Prakoso and Kulhawy [8] studied the reliability based design for rock footing. Stuedlein et al. [10]

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discussed about the probabilistic evaluation of undrained footing bearing performance using a second-moment probabilistic approach. The result of the study indicated that the reliable assessment of the spread footing response depends largely on the assumed strength anisotropy and soil layering. Also, Fan and Liang [7] studied the application of MCS on laterally loaded piles.

The present study aims at performing a reliability analysis of a RC isolated column footing using Monte Carlo simulations. The specific objectives of this study are: 1) to perform the reliability analysis of an isolated footing subjected to an axial load and the footing subjected to uniaxial moment,2) to find the variation of reliability index with load factor for each case, 3) to find the critical modes of failure for a given load factor and 4) to find how reliability is affected by the age of the footing.

2 STRUCTURAL REALIBILITY

Reliability is the probability that a system will perform its intended function over a specified period of time under specific operating conditions. Reliability analysis evaluates the probability of structural failure by determining whether the limit state functions are exceeded. Structural reliability aims at quantifying the probability of failure of systems due to uncertainties in their design, manufacturing and the environment conditions. The study of structural reliabilityis concerned with the calculation and prediction of the probability of limit-state violations at any stage during astructure's life. The probability of the occurrence of an event such as a limit-state violation is a numerical measure of the chance of its occurring. Once theprobability is determined, the next goal is to choose design alternatives that improve structural reliability and minimize the risk of failure [5].The reliability can be represented in the form of reliability index, which is given by 1- probability of failure. It is one of the best way to analyze the safety of the foundation and the structure.

In order to understand concept of reliability, consider a simply supported beam with deflectionexceeding the limiting deflection as one of the failure criteria. Let Y be the deflection of the beam and X be the limiting deflection. If Y>X, the structure will fail. In other word, the probability of failure is

P_f= P(Y>X)

If P(Y) is the probability density function of Y and X is assumed to be deterministic, the hatched portion in Fig.1 gives the probability of failure.

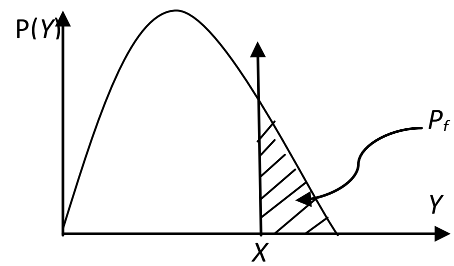


Fig.1: Probability of failure (Y is random and X is deterministic)

$$\beta = 1 - P_f$$

Where, β is the reliability index and P_f is the probability of failure.

3 MONTE CARLO SIMULATIONS

Monte Carlo methods were originally practiced under more generic names such as 'statistical sampling'. It is a simulation technique which has gained popularity after the electronic computers were first built. The first major use of Monte Carlo simulation was in 1944 in the research works to develop the first atom bomb [3, 6]. This technique has become popular in the fields of physics, physical chemistry and operations research in the 1950's [6].

In MCS, samples of data of some physical process or experiment are generated numerically without actually performing the test. All random variables are represented as ensembles of samples drawn from their respective distributions.A series of deterministic analysis is then performed to generate samples of output parameters, to compute the probabilistic quantities of interest. [4, 13].A schematic representation of Monte Carlo Simulation is shown in Fig. 2

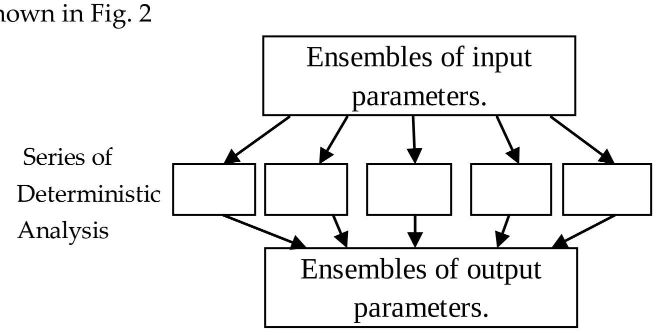
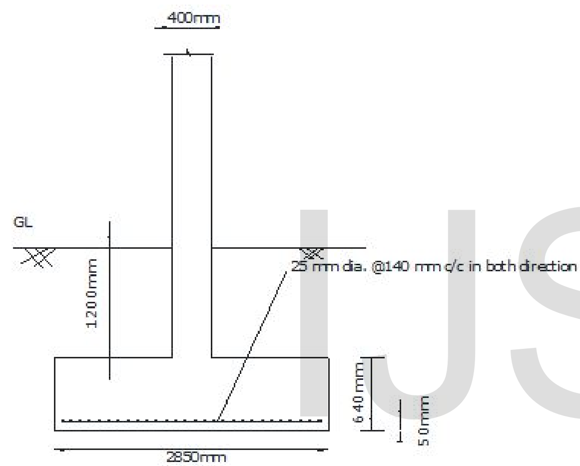


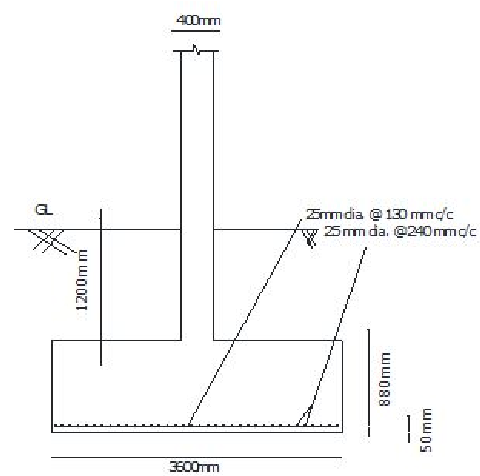
Fig. 2: Schematic representation of Monte Carlo simulation

4. RELIABILITY ANALYSIS OF ISOLATED FOOTINGS

In the present study, two isolated footings are considered. 1) A square footing designed for an axial load of 1800 kN and 2) A rectangular footing designed for an axial load (1800 kN) and uniaxial bending moment (60 kNm). M30 concrete and Fe415 steel are used for the footings. The safe bearing capacity (SBC) of soil was assumed as 250 kN/m2. The footings are designed as per IS: 456-2000. The details of the designed footings are given below in fig.3



(a)



(b)

Fig.3 Reinforcement details of Footings (a) square footing with axial load (b) Rectangular footing with axial load and uniaxial moment

4.1 Modelling of Uncertain Parameters

Axial load, compressive strength of concrete, proof stress of steel, ultimate bearing capacity (UBC), uniaxial moment and bar diameters are considered as normally distributed random variables. Dimensions of the footing (Length, breadth and depth) are considered as uniformly distributed random variables.

The parameters of the random variables considered in the analysis are provided in Tables 1 and 2.

Table1 Parameters of normally distributed random variables

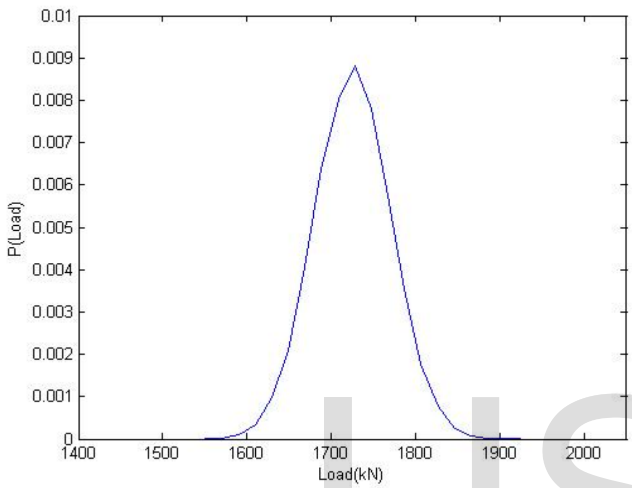
Random parameters	Mean	Standard deviation
Compressive strength of concrete(MPa)	38.25	5
Proof stress of steel (MPa)	415	10
Load (kN)	1725.25	45
Uniaxial moment (kNm)	60	6
Ultimate soil bearing capacity(kN/m²)	500	50
Diameter of bar(mm)	25	0.3

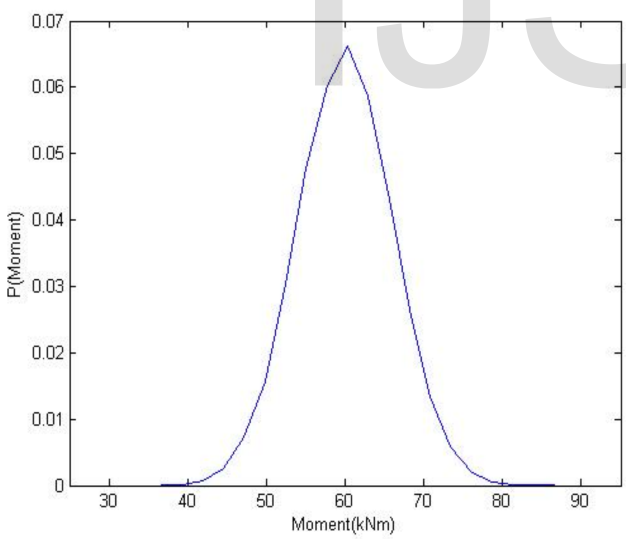
Table 2 Parameters of uniformly distributed random variables

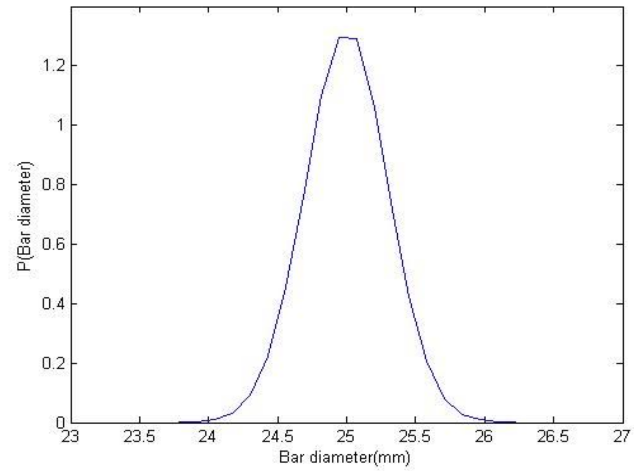
Random Parameters	Minimum value	Maximum value
Depth of square footing (mm)	630	650
Depth of rectangular footing (mm)	870	890
Length/Width of square	2.80	2.90

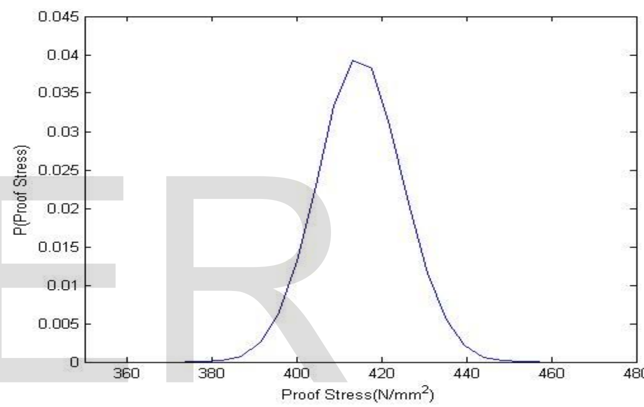
footing (m)		
Breadth of rectangular footing (m)	3.55	3.65
Length of rectangular footing (m)	2.35	2.45

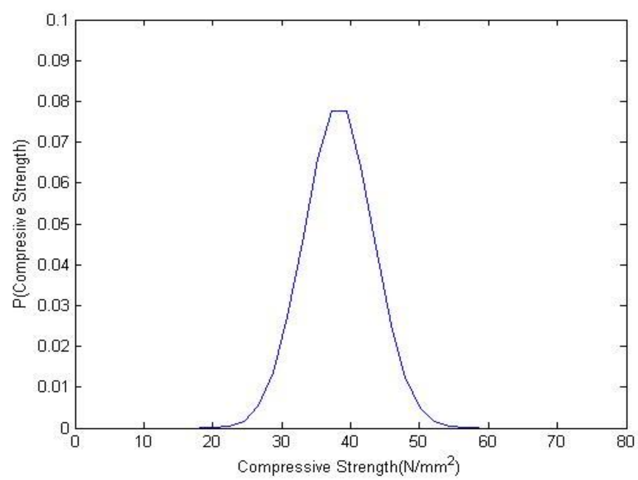
The pdf of the generated samples (with sample size of 100000) are shown in Fig. 4

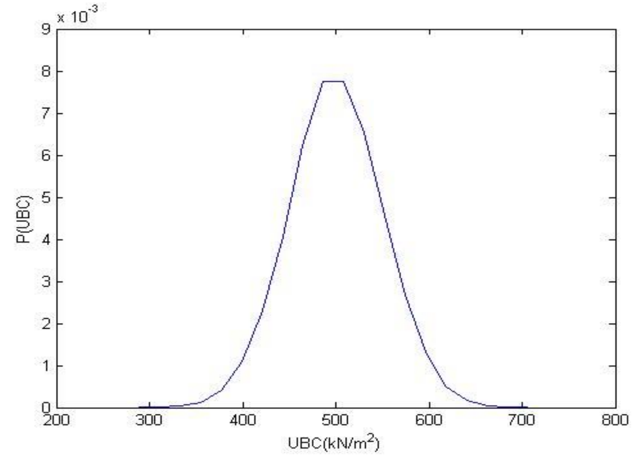


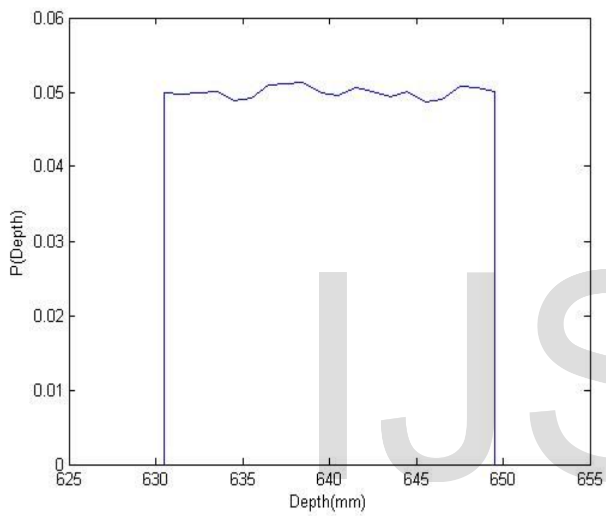


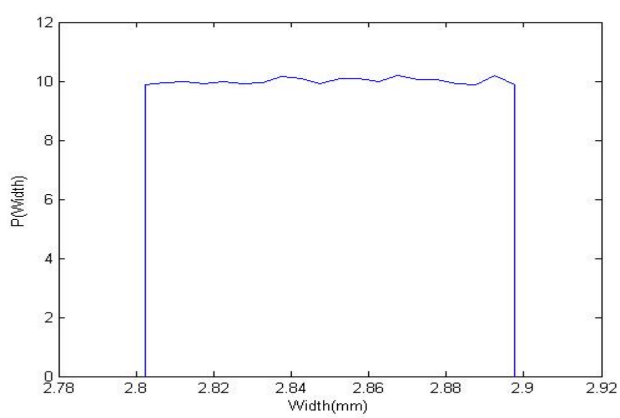












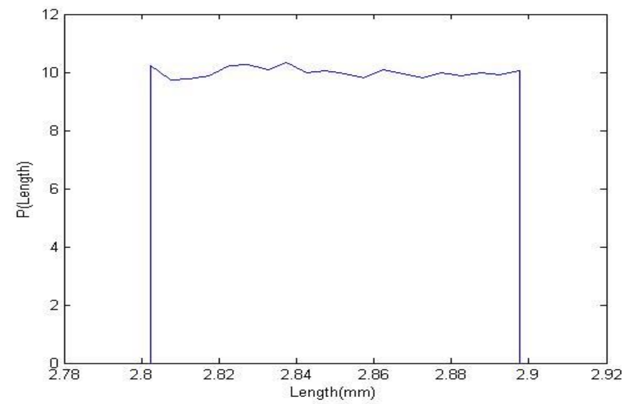


Fig.4: Probability density functions of some of the uncertain parameters

4.2 Failure Modes

The modes of failure considered in the present study are: 1) Bearing capacity failure, 2) one way shear failure, 3) punching shear failure, 4) bending failure and 5) bearing stress failure.

4.3 Monte Carlo Simulation

A sample size of N_s=100000 is used in the analysis. N_s samples of each of the random parameters are generated from the respective distributions. Analysis is carried out using codes developed in Matlab. To analyze failure, bending resistance and shear resistance of the footing are calculated without considering the partial safety factors. Total instances of failure and failure counts for each of the failure modes are noted. If N_s is the number of samples and C is the failure count then $P_f = \frac{C}{N_S}$ and reliability index is givenby $\beta = 1 - P_f$.

4.4 Effect of Age of Strength Parameters of Footing

It would be useful to study how reliability of footings vary with age. Though one can expect an increase in compressive strength of concrete with age, the reinforcement in the footing may corrode leading to reduction in its effective cross sectional area. The variation in strength of concrete has been studied by MacGregor [14]. The compressive strength, $f_c(t)$, of M30 concrete is given by:

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$$f_c(t) = \begin{cases} 15 \cdot 85 + 4.03\ln(t)\text{MPa}, t < 10\ yrs \\ 48 \cdot 9\ \text{MPa}, t \geq 10\ \text{yrs} \end{cases} \quad (3)$$

Here, $t$is in days. When cracks develop in concrete, it may lead to exposure of reinforcement to moisture causing corrosion. The carbonation transformation process, or spread of chlorides inside concrete, is the main reason for breaking the passive protection of steel reinforcement and thus causing the beginning of corrosion. The corrosion rate increases significantly when the relative humidity increases. The corrosion rate is reported to be within 0.015-0.09mm/year [15]. In this study, corrosion rate is assumed as 0.05mm/year.

5. RESULTS AND DISCUSSION

5.1 Variation of Reliability Index with load factor

In limit state method, design is carried out by magnifying the characteristic load by a load factor of 1.5. The present study investigates the reliability of the foundation designed using the limit state method. It is also interesting to study the reduction of reliability index with increased load factors. Fig. 5 shows the variation of reliability index with load factor of square footing.

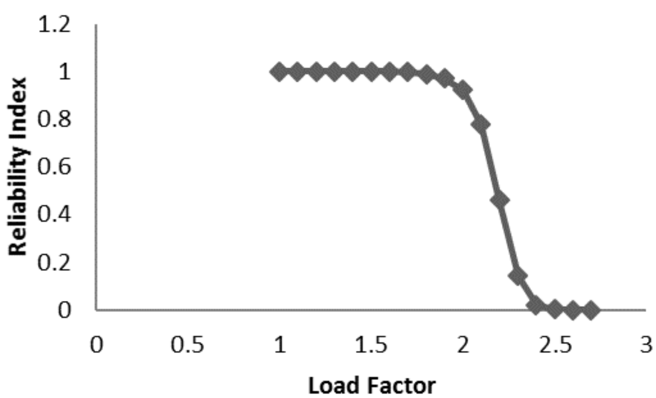

Fig.5: Variation of Reliability index with load factor for isolated footing subjected to axial load

The reliability index corresponding to load factor 1.50 is found to be 0.9998. This means that the probability of failure is 2 in 10000. The reliability index corresponding to load factors 1.2, 1.5, 1.8 and 2.0 are found to be 1, 0.9998, 0.9892 and 0.9214 respectively. Since, probability of failure increases with increase in load factor, same structure may not be reliable when the load is increased either by adding floors or by changing the occupancy of the building which induces higher loads than designed. From fig.5, it is clear that the reliability decreases with increase in load factor and the chances of failure is 100% as load factor approaches 2.5.

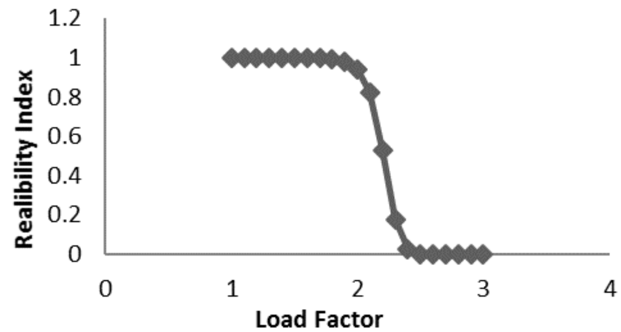

Fig.6: Variation of Reliability index with load factor for the isolated footing with axial load and uni-axial moment

The variation of reliability index with load factor for an isolated rectangular footing with axial load and uniaxial moment is shown in fig.6.From, the graph, it is clear that the reliability decreases with increase in load and hence the probability of failure increases. The reliability corresponding to load factor 1.50 is 0.9998 and at a load factor of 2.20, it suddenly reduces to 0.5083. Reliability based design can be done by selecting a suitable load factor for the target reliability.

5.2 Critical Mode of Failure corresponding to different load factors

The % failure of the footing under different failure modes is shown in fig. 7. In the figure 1, 2,3, 4 and 5 represent bearing capacity failure, one way shear, two way shear, bearing stress and bending failure respectively. Failure corresponding to load factor 1.50 is almost nil whereas that for 1.80, the critical mode of failure is bearing capacity failure with 1.08% failure. For, load factor 1.20, the critical mode is bearing capacity failure with 7.197% chance of failure. In the present study, SBC is calculated from UBC using a factor of safety of 2. The obtained result indicates that SBC should be estimated with sufficient factor of safety, depending on the uncertainty associated with the estimated bearing capacity from the field investigations.

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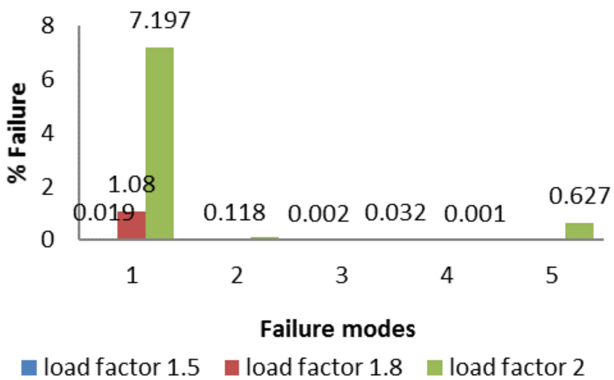

Fig 7: % failure of different modes of theisolated footing subjectedto axial load.

The % failure under different modes for footing subjected to axial load and uni-axial moment is shown in fig. 8.Here 1, 2, 3, 4, 5, 6 and 7 marked in x-axis represent different failure modes - bearing capacity, one way shear along both directions, bending stress, bending failure along both directions and punching shear respectively. Critical failure modes at load factors 1.80 and 2.0 is bearing capacity with 0.76% and 5.334% chances of failure whereas for 1.50, it is almost nil.

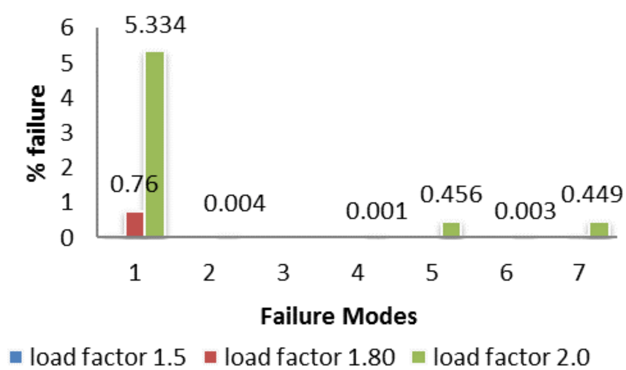

fig.8: % failure of different modes of the isolated footing subjectedto axial load and uni-axial moment

5.3 Effect of Age on Reliability

The variation of reliability with age for the two footings considered in the study is shown in fig 9 and 10.

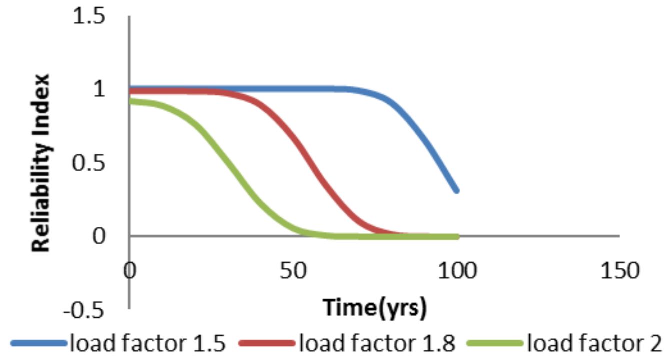

Fig. 9: Effect of age on Reliability of the isolated footing with axial load

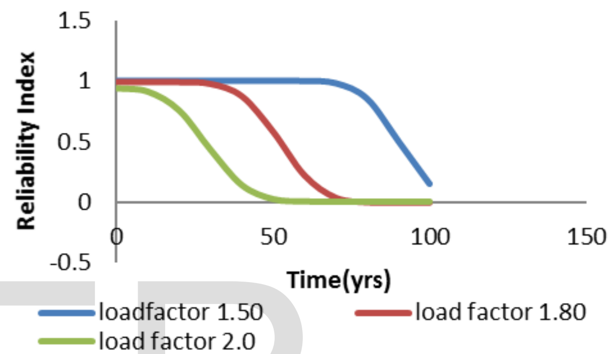

Fig.10: Effect of age on Reliability of isolated footing subjected to axial load and uni-axial moment

From the figures, it is clear that the reliability of the structure decreases with time after a certain period. For load factor 1.50, foundation is reliable for almost 75 years whereas for 1.80 and 2.0, it reduces due to the effect of corrosion in steel. The reduction in area of steel as a result of corrosion is balanced by the increased strength of concrete. Deterioration of the concrete with age is not considered in the present study.

6. CONCLUSIONS

Monte Carlo simulation provides an effective method to take into consideration the effect of uncertainties in engineering problems. Civil engineering structures are designed in a deterministic frame work, even though majority of the parameters involved are random in nature. Reliability analysis of isolated footings is carried out in the present study. The effect of load factor on reliability is investigated. Knowing the variation of reliability index with load factor, suitable load factor can be chosen by the designer to achieve a given reliability. The % failure of different failure modes were also studied. The bearing capacity failure was found to be critical in the present investigation. The variation of reliability index with age is also studied. For structures designed with load factor of 1.5, reliability is not found affected during the initial periods, probably due to the strength gain in concrete with age. Later, corrosion in steel and the subsequent reduction in the effective steel area results a decrease in the reliability index. The present study demonstrates the usefulness of Monte Carlo Simulation in reliability analysis of structural systems.

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